The Effect of Movement Behavior on Population Density in Fragmented Landscapes

Nazanin Zaker, Laurence Ketchemen, under the supervision of Dr. Frithjof Lutscher

Faculty of Science, University of Ottawa

Research Questions

Landscape fragmentation arises from human activities and natural events, and may create abrupt transitions ('interface') in landscape quality. Thus, we face with the following questions in theoretical ecology:

- How does landscape structure affect population distribution in space?
- What effect does movement in response to habitat variation have on population density?

The Model

The mathematical model is:

$$\begin{cases} u_{i,t} = D_i u_{i,xx} + r_i u_i \left(1 - \frac{u_i}{K_i}\right), & (x,t) \in \Omega_i \times [0,\infty) \\ D_1 u_{1,x}(0,t) = D_2 u_{2,x}(0,t), & t \geq 0 \\ u_1(0,t) = k u_2(0,t), & t \geq 0 \\ u_{1,x}(-L_1,t) = u_{2,x}(L_2,t) = 0, & t \geq 0 \end{cases} \qquad (1)$$

where

- $x \in \Omega_i$ location in each patch
- $t \geq 0$ time
- $u_i(x,t)$ population density
- $D_i > 0$ diffusion coefficient
- $r_i > 0$ growth rate
- $p_i > 0$ habitat preference ($p_1 + p_2 = 1$)
- $K_i > 0$ carrying capacity
- $k = \frac{p_1 D_2}{p_2 D_1}$ discontinuity 'jump' at the interface

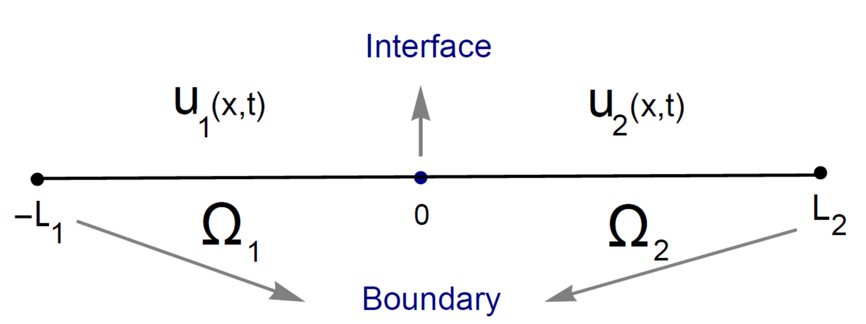


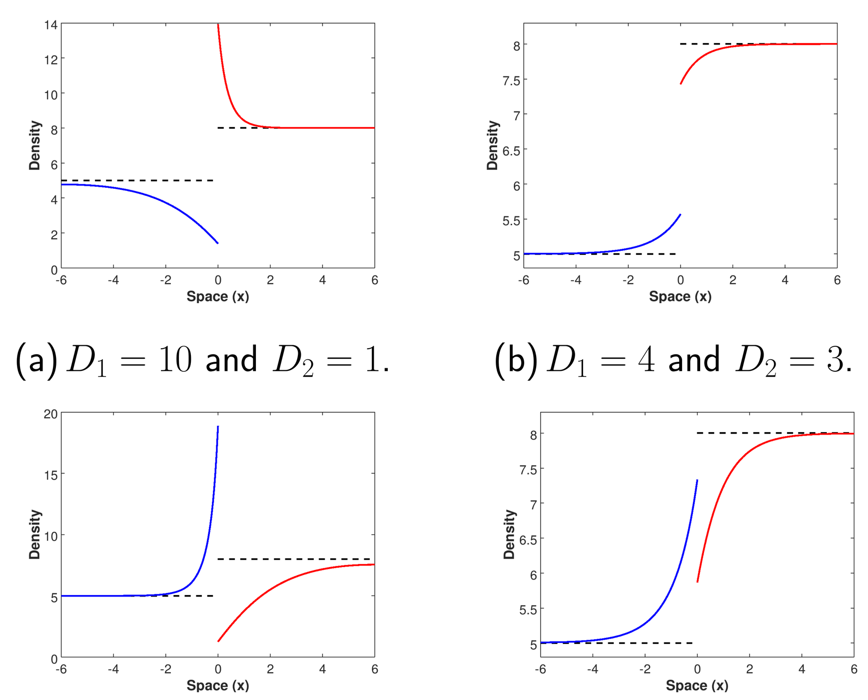
Fig. 1: Model set-up

Uniqueness and global existence of solutions of this time-dependent problem were recently proved [3].

Existence of the Steady State

Theorem: *The unique positive steady-state solution of (1) exists and it is globally asymptotically stable.*

Illustration of the steady-state profile:



(a) $D_1 = 10$ and $D_2 = 1$. (b) $D_1 = 4$ and $D_2 = 3$.

(c) $D_1 = 1$ and $D_2 = 15$. (d) $D_1 = 4$ and $D_2 = 5$.

Fig. 2: Effect of diffusion rate on the steady state

Property of the Steady State

Question: When does the total density (TD) exceed the total carrying capacity (TC)? [1][2]

Proposition: *If $K_2 > K_1$ and*

$$\frac{K_1}{K_2} < k < \frac{K_1 r_2}{K_2 r_1}, \qquad (2)$$

then the property

$$\text{TD} = \int_\Omega u(x)\mathrm{d}x > \int_\Omega K(x)\mathrm{d}x = \text{TC} \qquad (3)$$

holds.

Condition (2) is sufficient but not necessary:

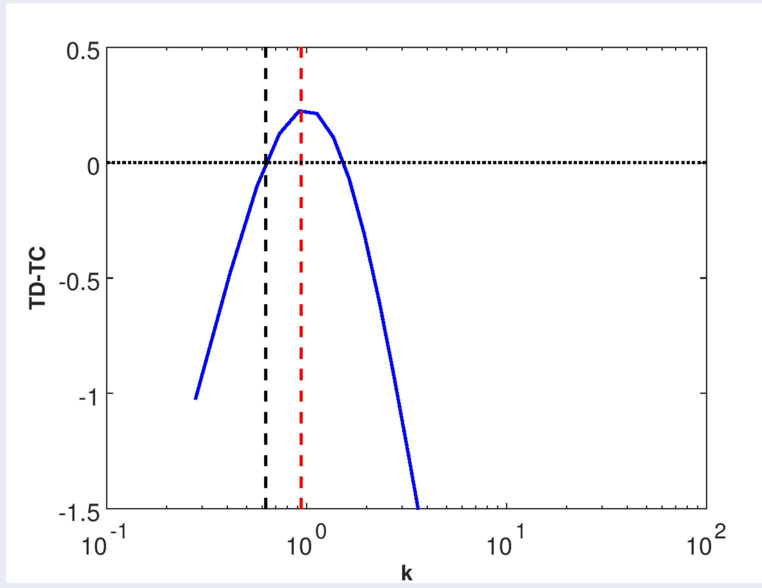


Fig. 3: TD−TC as a function of $\log(k)$. The vertical black and red dashed lines correspond to bounds in (2).

Numerical Results

We numerically evaluate property (3) for the four plots in Figure (2).

Fig 2	TD	TC	TD−TC
(a)	73.531	78	-4.4691
(b)	78.148	78	0.14769
(c)	70.292	78	-7.7085
(d)	78.136	78	0.13557

Table 1: Numerical evaluation of (3) at the steady state

Figure (2) and Table (1) show that:

- Case (b) satisfies condition (2) since $1 < kK_2/K_1 = 1.2 < 1.5 = r_2/r_1$
- Case (d) does not satisfy condition (2) since $kK_2/K_1 = 2 > 1.5 = r_2/r_1 > 1$
- Property (3) is satisfied for both cases (b) and (d)

The Limits of Fast and Slow Diffusion

The effect of movement behavior as D_i goes to zero or infinity with fixed ratio $d = \frac{D_1}{D_2} = 0.9$

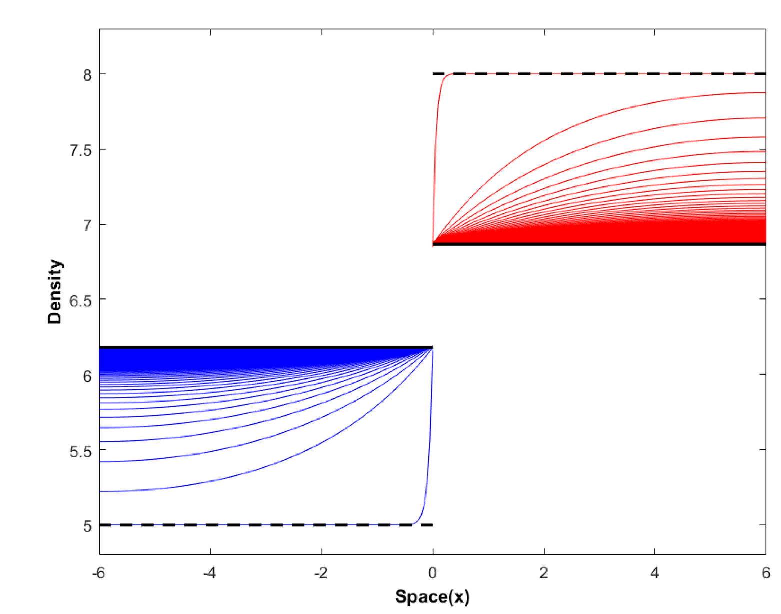


Fig. 4: Steady-state profiles when varying D_1

- As $D_1 \to 0$:
  - The steady-state density approaches the carrying capacity on each patch (indicated by the black dashed lines).
  - The steady state has an increasingly steep and narrow transition zone near the interface (the lowest of the blue curves and the highest of the red curves).
- As $D_1 \to \infty$:
  - The steady-state solution becomes increasingly flat and approaches constant values u_i^* on patch i (indicated by the black solid lines).
  - The limiting values for large D_i are

$$u_2^* = \frac{L_1 r_1 k + L_2 r_2}{\frac{L_1 r_1 k}{K_1/k} + \frac{L_2 r_2}{K_2}}, \quad u_1^* = k u_2^*. \qquad (4)$$

Summary

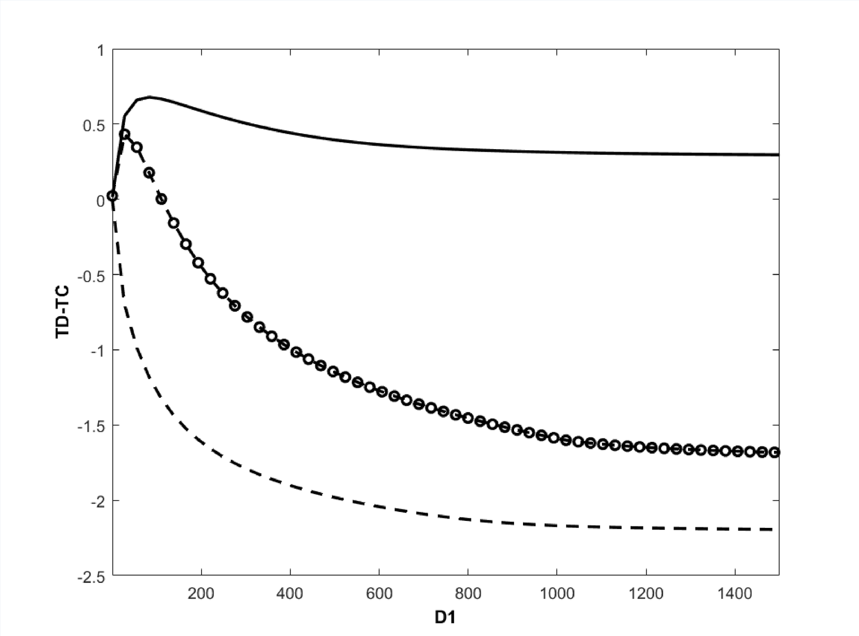


Fig. 5: TD−TC versus D_1 for three values of k

1. When $k < K_1/K_2$, property (3) does not hold for any value of D_1 (dashed curve).
2. When condition (2) is satisfied, property (3) holds for all D_1 (solid curve). The curve has an intermediate maximum.
3. When $k > (K_1 r_2)/(K_2 r_1)$ is not too large, the curve increases, initially but decreases below zero eventually (dashes and circles).
4. When $k \gg (K_1 r_2)/(K_2 r_1)$, the curve has the same shape as for small k: it is monotone decreasing (plot not shown).
5. The limiting value for all the above cases for large D_i is given by (4).

References

[1] D. DeAngelis et al. (2016) *JMB*

[2] Y. Lou (2006) *JDE*

[3] G. Maciel et al. (2019) *JMB*